Statistical properties of breaking waves in field conditions. A gaussian field approach

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Motivation

Breaking waves play significant role for various ocean and atmosphere processes starting from wind speeds 5-7 m/s

- dissipation of wind wave energy
- ocean-atmosphere interaction, e.g. momentum, heat, moisture and gas exchange
- dynamics of upper ocean layer, turbulent mixing
- sea surface roughness, acoustical, optical and dielectric properties; remote sensing applications

Modeling of breaking waves

There are two main approaches for modeling of processes associated with breaking waves:

Empirical parametrization is the most popular approach to describe dependence of whitecap statistics from external parameter (e.g. whitecap coverage vs. wind speed or wave age).

However, existing experimental datasets cannot be described by a function of a single external parameter.

Modeling of breaking waves

Spectral representation and Phillips's Λ-function.

Various key-properties of wave breaking statistics can be estimated through spectral density of breaking crest length per unit area, $\Lambda(\mathbf{c})=\Sigma\, l(\mathbf{c})$ (where **c** is the vector phase velocity).

Spectral description of breaking waves in connection with it's cinematics makes Λ(c) function to be powerful method of experimental, theoretical and applied investigations.

But Λ(c) function remains a space-time averaged characteristic and cannot be directly applied for the description or modeling of properties of naturally observed breaking wave field.

Objectives

Many taks (e.g. high-resolution models, remote sensing applications) require more detailed description of breaking field.

For the realistic description of whitecap field we need to know detailed statistics of breaking waves in each spectral interval.

Particularly, we need to find a way to describe geometry of breakers, distribution of breaking crest length and breaker surface sizes in connection with properties of wave field.

Mathematical Framework

Sea surface waves with phase velocity in the range of **c**, **c**+d**c** and propagating in direction θ is considered as a stationary, homogeneous, two-dimensional Gaussian random field with mean zero $\overline{X(\boldsymbol{\xi})} = 0$

Suppose a_{cr} is the critical amplitude when wave breaking occurs

Number of up-crossings of critical amplitude on the unit interval is given by (Adler [1981])

$$\lambda = \frac{1}{2\pi}\sqrt{\frac{k''(0)}{k(0)}}\exp\left(-\frac{a_{cr}^2}{k(0)}\right)$$

where *k* is the covariance function

High-level DISTRIBUTION and PoisSon process

If a_{cr} is sufficiently high when distribution of maximums $X_{a_{cr}} = X(\boldsymbol{\xi}) > a_{cr}$ is close to Poisson process with rate λ

For such high levels, zones above this level tend to be composed of disjoint convex sets which are sections of elliptic paraboloid with elliptic footprint $z = a_{cr}$ lying in the plane and with surfaces distributed exponentially p(S)~Exp(S)

High-level DISTRIBUTION and PoisSon process

Gaussian random field **X** which takes sufficiently high level **u** can be presented in form $X = u + \frac{u}{2}C + o(u^{-1})$ where ***C*** is the normalized matrix of second order spectral moments.

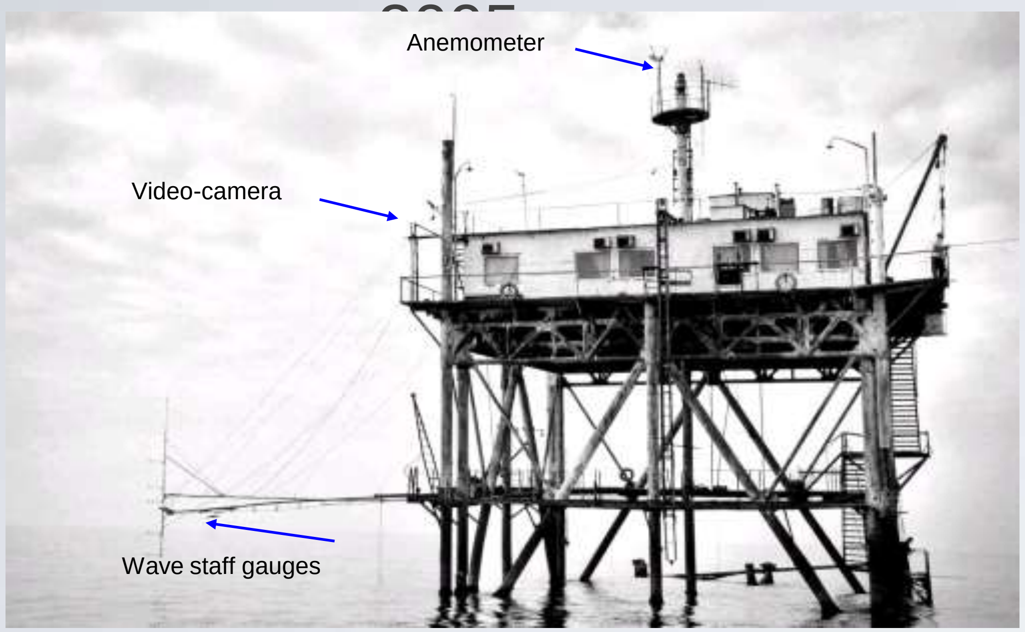
With ***k*** in the squared exponential form, correlation function for the breaking crest can be expressed as

$$k = k(0) - \frac{x^2}{2b^2} \; or \; k = k(0) - \frac{x^2 mss_x}{2}$$

So that $mms_x = \frac{u}{k(0)b^2}$

Experimental DATA SET

Black Sea Oceanographic Platform. Years 2003 and 2005



Video observation of whitecaps

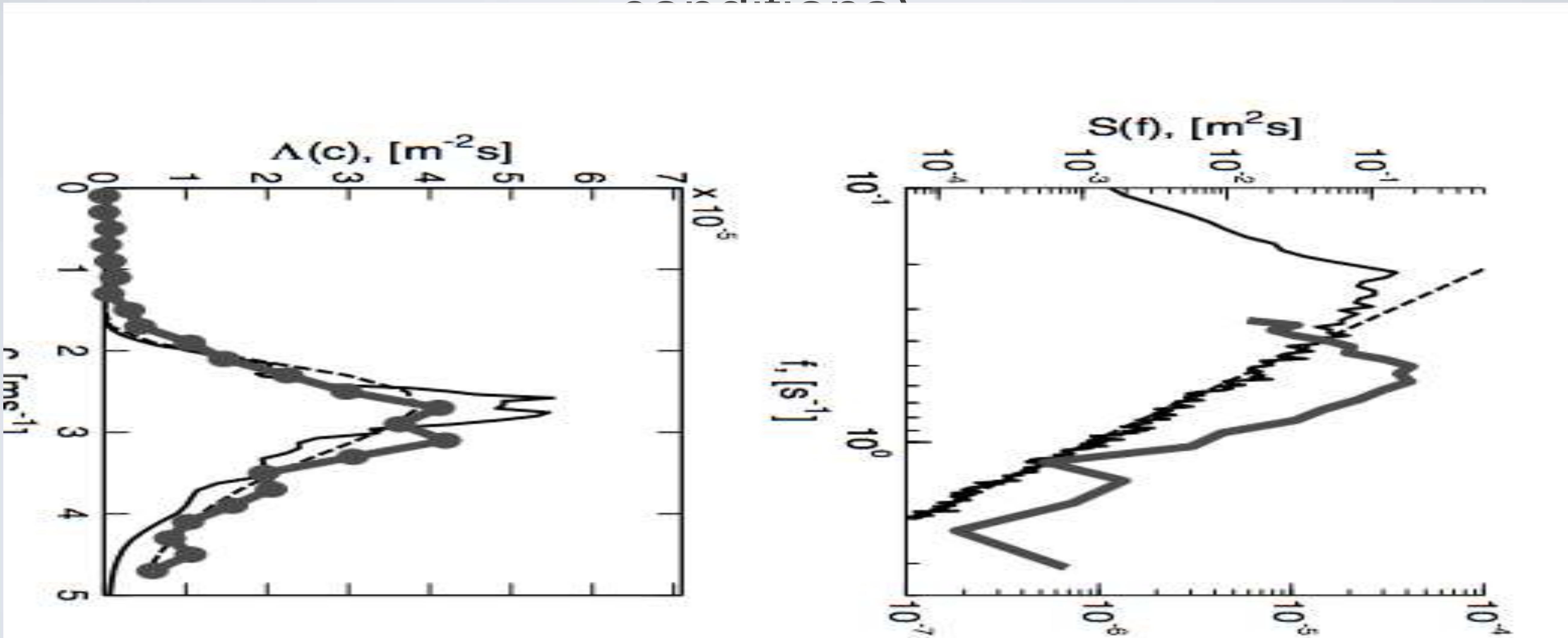
Mironov et al. 2008

Selected cases (completely different situations)

Case num.	U_{10} Wind speed, m/s	C_p Peak phase velocity, m/s	Wave age, U_{10}/C_p
1	15	8.7	1.8
2	6	4.4	1.6
3	11	2.4	4.6

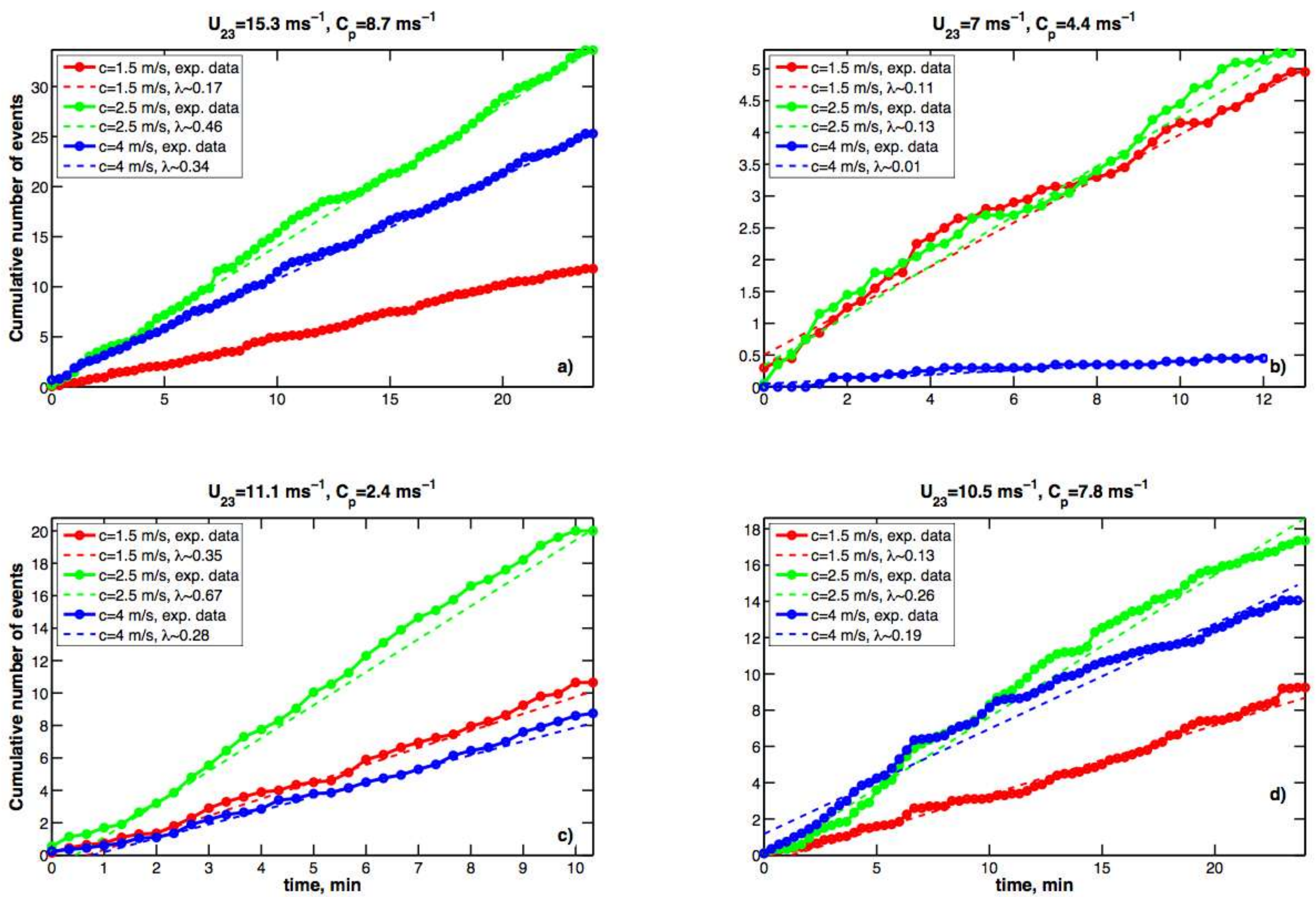
Experimental Results

Lambda functions for two different experimental situations (left - developed sea; right - short fetch wave conditions)



Poisson distribution

Experimental validation of Poisson process

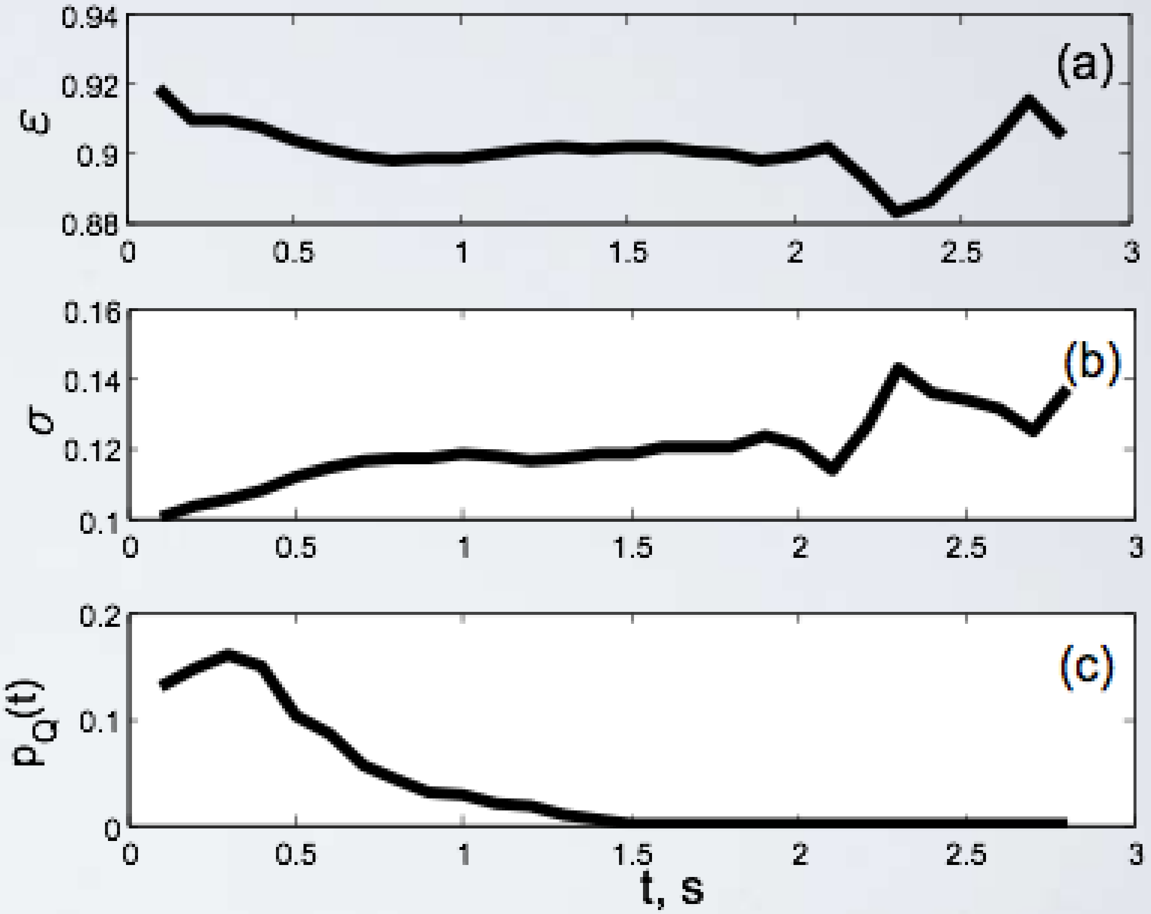


Self-similarity OF breaking crests

Each whitecap can be approximated with ellipse. Eccentricity of approximation ellipse can be used as measure of similarity of observed breakers

$$\varepsilon = \sqrt{1 - \frac{a^2}{b^2}} \approx 0.9$$

Time evolution of averaged eccentricity for all registered whitecaps (over 200 000 individual events)



(a) time evolution of average eccentricity
(b) standart deviation for each time domain
(c) impact of time interval into totally observed whitecap coverage

Exponential DISTRIBUTION of sizes of observed breakers

For high-level restricted process according to Adler [1981] mean perimeter of intersection of restricting plane with Gaussian field is

$$\overline{l_{a_{cr}}} = 2\lambda/2 = \frac{1}{2\pi}\sqrt{\frac{k''(0)}{k(0)}}\exp\left(-\frac{a_{cr}^2}{k(0)}\right)$$

Surface of the breaker is proportional to square of the perimeter

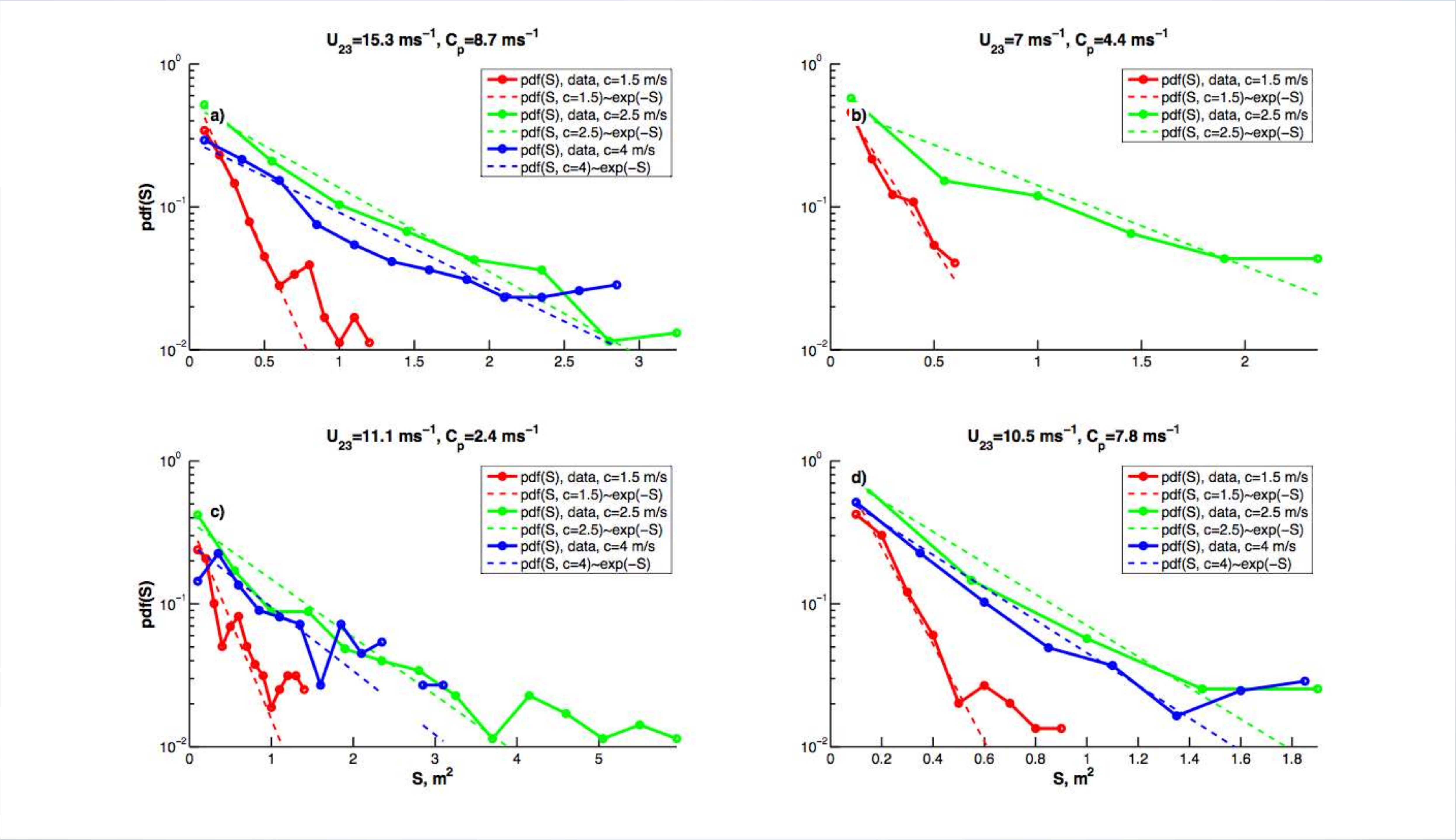
$$S \approx 0.015 l_{a_{cr}}^2$$

For exponentially distributed sizes of breaking wave we have

$$p(S) = \frac{\sqrt{2}}{\sqrt{\lambda}}\exp\left(-\frac{a_{cr}\sqrt{2}}{\sqrt{\lambda}}\right)$$

Exponential DISTRIBUTION of sizes of observed breakers

Experimentally obtained size distribution of breaking



Distribution OF Breaking lengths

For self-similar breakers, breaking crest length is proportional to square root of whitecap surface. Square root of exponentially distributed process represents a Rayleigh distribution:

$$\text{If } X \text{ is } Exp(\sqrt{\lambda}/\sqrt{2}), \text{ then}$$

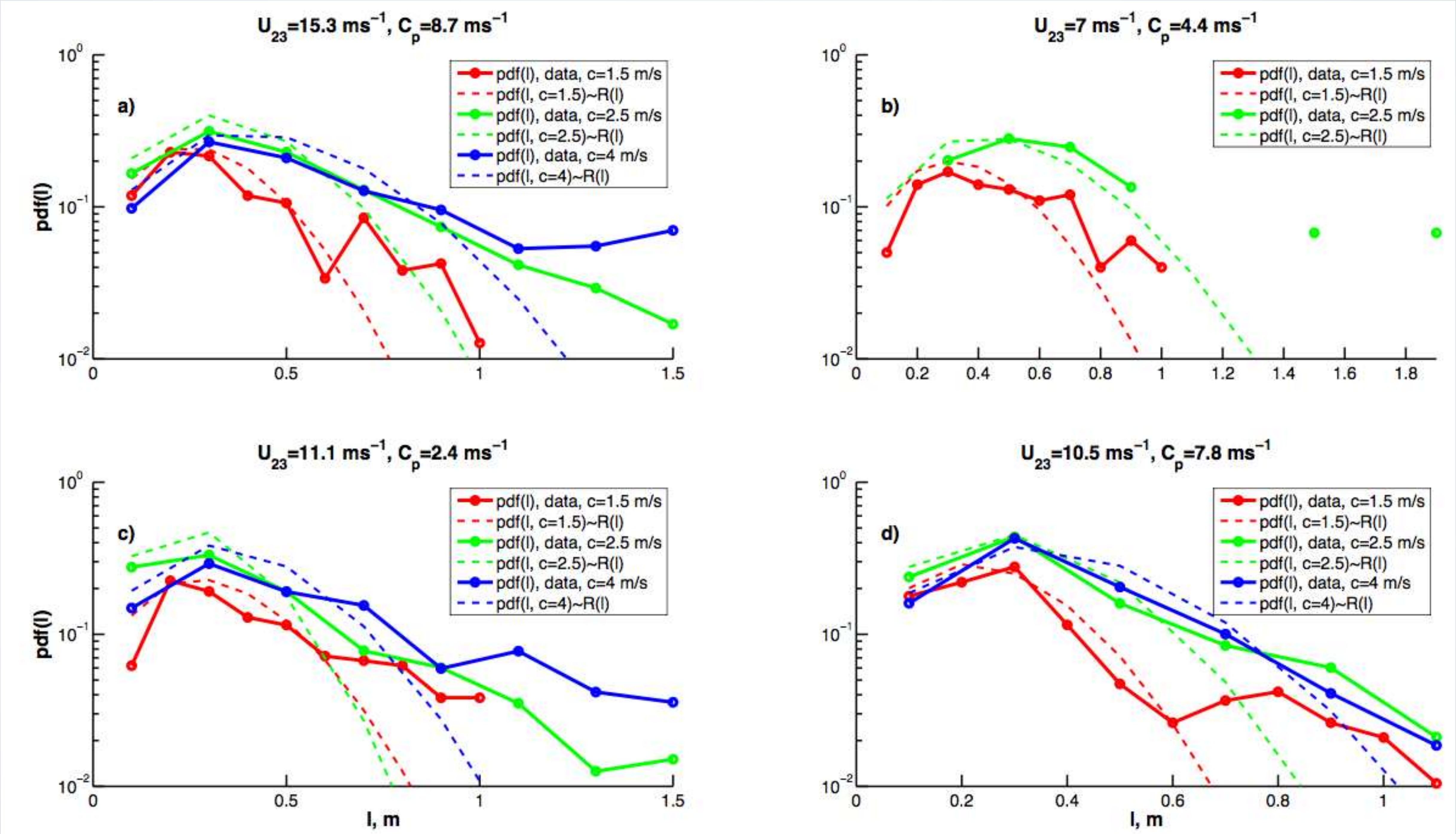
$$Y \propto 0.6\sqrt{X} \sim Rayleigh(\sigma = \frac{0.6\sqrt{2}\lambda^{1/4}}{2^{1/4}\sqrt{\pi}})$$

Finally, for the distribution of breaking crests we have:

$$p(l) = \frac{\pi\sqrt{2}l}{0.6^2 2\sqrt{\lambda}} \exp\left(-\frac{\pi\sqrt{2}l^2}{0.6^2 4\sqrt{\lambda}}\right)$$

Distribution OF Breaking lengths

Experimental surface size distribution of breaking waves



Results

We propose new approach which is aimed to describe statistics of individual breaking waves in field conditions. Assuming that sea surface elevation field is close to the Gaussian we have:

- Wavebreaking statistics in each isolated spectral and directional interval represents Poisson process as it should be for high-level distributions of Gaussian field
- Whitecaps can be represented as self-similar ellipses with exponentially distributed surface sizes
- In the present framework self-similarity of experimentally observed breaking events means that relation between components of *mss* of breaking waves is always constant
- Breaking crest lengths are Rayleigh distributed